updated March 28, 2022

1 332 Problems

Notes:

- If you use MATLAB or Python to solve any of these problems, include the code that you used.

1.1 Course Organization

1) Send an email to Prof. McCue (ian.mccue@northwestern.edu), Qihua Chen (r6p5t6@u.northwestern.edu), and Donghoon Shin (donghoonshin2023@u.northwestern.edu). If you have not taken CE 216 or the MSE 316 sequence, let us know why. Otherwise, let us know how comfortable you feel with the material from CE 216 and MSE 316-1. Also let us know what you have enjoyed most about MSE so far, what you like least (if such a thing exists!) and if you are involved in any research within the department or elsewhere.

1.2 The Stress Tensor

2) Consider the following stress tensor:

$$\sigma_{ij} = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & 2 \\ 0 & 2 & 6 \end{bmatrix} x10^6\,\mathrm{Pa}$$

(a) Calculate the stress tensor for coordinate axes rotated by 30° about the z axis (the 3 axis).

(b) Repeat the calculation for a 30° rotation around the x axis (the 1 axis).

(c) Calculate the three principal stresses.

(d) Calculate the maximum shear stress in the sample.

3) Consider the following stress tensor:

$$\sigma_{ij} = \begin{bmatrix} -2 & 1.4 & 0 \\ 1.4 & 6 & 0 \\ 0 & 0 & 2 \end{bmatrix} x10^6\ \mathrm{Pa}$$

(a) Draw a Mohr circle representation of the stress contributions in the xy plane

(b) What are the three principal stresses?

(c) Show that the the sum of the diagonal components from original stress tensor is equal to the sum of the three principal stresses. What is the hydrostatic pressure for this stress state?

1.3 Strains

4) An engineering shear strain of 1 (100%) is applied to a rubber cube with dimensions of 1cm×1cm×1cm. Young's modulus for the rubber sample is 10^6 Pa, and you can assume it is incompressible.

(a) Sketch the shape of the object after the strain is applied, indicating the dimensions quantitatively.

(b) On your sketch, indicate the magnitude and directions of the forces that are applied to the object in order to reach the desired strain.

(c) Calculate the 3 principal extension ratios characterizing the final strain state.

1.4 Typical Moduli

5) Calculate the sound velocities for shear and longitudinal waves traveling through the materials listed in the 'Representative Moduli' table from the course text.

1.5 Matrix Representation of Stress and Strains

6) For an isotropic system there are only two independent elastic constants, s_{12} and s_{11}. This is because if properties are isotropic in the 1-2 plane, the compliance coefficient describing shear in this plane,s_{44}, is equal to$2\left(s_{11}-s_{12}\right)$. We can use the Mohr's circle construction to figure out why this equality must exist.

(a) Start with the following pure shear stress and strain states:

$$\sigma=\begin{bmatrix}0 & \sigma_{12} & 0\\ \sigma_{12} & 0 & 0\\ 0 & 0 & 0\end{bmatrix};\ e=\begin{bmatrix}0 & e_{12} & 0\\ e_{12} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Use the matrix formulation to obtain a relationship between σ_{12} and e_{12} in this coordinate system.

(b) Rotate the coordinate system by 45° so that the stress state looks like this:

$$\sigma=\begin{bmatrix}\sigma_1^p & 0 & 0\\ 0 & \sigma_2^p & 0\\ 0 & 0 & 0\end{bmatrix};\ e=\begin{bmatrix}e_1^p & 0 & 0\\ 0 & e_2^p & 0\\ 0 & 0 & 0\end{bmatrix}$$

Use the Mohr's circle construction to write these principal stresses and strains in terms of σ_{12} and e_{12}. Then use the matrix formulation to obtain an expression betweenσ_{12} ande_{12} in this rotated coordinate system.

(c) For an isotropic system, the relationship between σ_{12} ande_{12} should not depend on the orientation of the coordinate axes. Show that the only to reconcile the results from parts a and b is for $s_{44}$to be equal to $2\left(s_{11}-s_{12}\right)$.

7) Consider a material with orthorhombic symmetry, with different properties along the 1, 2 and 3 directions. Young's moduli are measured along the 3 different directions, and we obtain the following results:
$E_1 = 5.5$ GPa; $E_2 = 2.0$ GPa; E_3=3GPa

(a) Is this material a metal, a ceramic or a polymer? How do you know?

(b) The compliance matrix, s, is a symmetric 6x6 matrix as shown below. For this material, cross out all of the elements that must be zero.

$$\begin{bmatrix}s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16}\\ s_{12} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26}\\ s_{13} & s_{23} & s_{33} & s_{34} & s_{35} & s_{36}\\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45} & s_{46}\\ s_{15} & s_{25} & s_{35} & s_{45} & s_{55} & s_{56}\\ s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & s_{66}\end{bmatrix}$$

(c) What are the values of s_{11}, s_{22} and s_{33} for this material?

(d) A value of 0.38 is obtained for Poisson's ratio is measured in the 1-2 plane by applying a tensile stress in the 1 direction and measuring the strains in the 2 direction. What is the value of s_{12} for this material?

8) Consider a material with elastic constants given by the following compliance matrix:

$$s_{ij} = \begin{bmatrix} 14.5 & -4.78 & -0.019 & 0 & 0 & 0 \\ -4.78 & 11.7 & -0.062 & 0 & 0 & 0 \\ -0.019 & -0.062 & 0.317 & 0 & 0 & 0 \\ 0 & 0 & 0 & 31.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 61.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 27.6 \end{bmatrix} \text{GPa}^{-1}$$

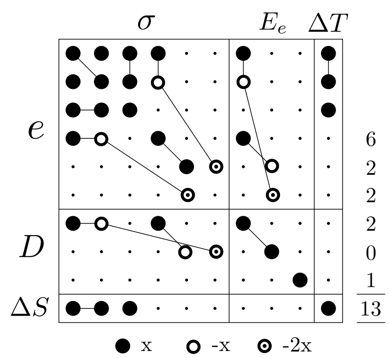
(a) Describe the symmetry of this material, and explain your reasoning.

(b) What is the highest value for Young's modulus that you would expect for this material? What direction does it correspond to?

(c) Calculate the value of Poisson's ratio obtained from an experiment where the materials is pulled in the 3 direction and change in sample width in the 2 direction is measured.

1.6 Other Linear Properties

9) Quartz has the 32 point group which has a coefficient map that looks like this:



(a) Consider the converse piezoelectric effect, where an electric field is applied along a particular direction and we are interested in determining the resulting strains in the material. Will any strains be generated in the material if I apply an electric field along the '3' direction of the crystal? Why or why not?

(b) Compare the normal strains in the 1 and 2 directions that are obtained when an electric field is applied in the 1 direction.

(c) Are any shear strains developed in the material when an electric field is applied in the 1 direction? If so describe the orientation of this shear strain.

(d) How many independent elastic constants are there for quartz?

(e) Does the shape of a chunk of single crystal quartz change when you heat it up? (Note: this is another way of asking if the thermal expansion of quartz is isotropic).

1.7 Yield Criteria

10) A cube of material is loaded triaxially resulting in the following principal stresses at the point of plastic yielding: σ_1^p =140 MPa, σ_2^p =20 MPa, and σ_3^p =35 MPa.

(a) What is the shear strength of the material according to the Tresca yield criterion?

(b) If the the value of σ_3^p were increased to 70 MPa, how does this change your result? Explain.

11) From the work of D. C. Jillson, Trans. AIME 188, 1129 (1950), the following data were taken relating to the deformation of zinc single crystals:

Table 1.1: Applied tensile force for slip of single crystal Zn.

ϕ	λ	P (N)
83.5	18	203.1
70.5	29	77.1
60	30.5	51.7
50	40	45.1
29	62.5	54.9
13	78	109.0
4	86	318.5

In this table ϕ is the angle between the loading axis and the normal to the slip plane,λ is the angle between the loading axis and the slip direction and$P$ is the force acting on the crystal when yielding begins. The crystals have a cross-sectional area,A_0, of 122x10^{-6}m^2.

(a) What is the slip system for this material.

(b) For each combination of P, ϕ and λ, calculate the resolved shear stress, τ_{RSS} and normal stress, σ_N acting on the slip plane when yielding begins.

(c) From your calculations, does τ_{RSS} or σ_N control yielding?

(d) Plot the Schmid factor versus the applied stress, P/A_0, acting on the rod. At what Schmid factor value are these experimentally-measured yield loads at a minimum? Does this make sense?

12) The tensile yield stress of a materials is measured as 45 MPa by a uniaxial tensile test.

(a) What will the shear stress of the material by if the materials yields at a specified value of the Tresca stress?

(b) Now calculate the same quantity (shear yield stress) if the material yields at a specified value of the Von Mises stress.

(c) Suppose the material is a glassy polymer like Plexiglas, and Tresca yield stress is obtained from a uniaxial compression experiment and from a uniaxial tensile experiment. Which of these experiments to you expect to give the largest Tresca yield stress?

13) What is the effect of the resolved normal stress on the yield behavior of crystalline metals and ceramics? What about polymers? Describe any differences between the two cases.

14) A sample of pure iron has a uniaxial tensile yield strength of 100 MPa. Assume that the yield behavior is described by the Von Mises yield criterion.

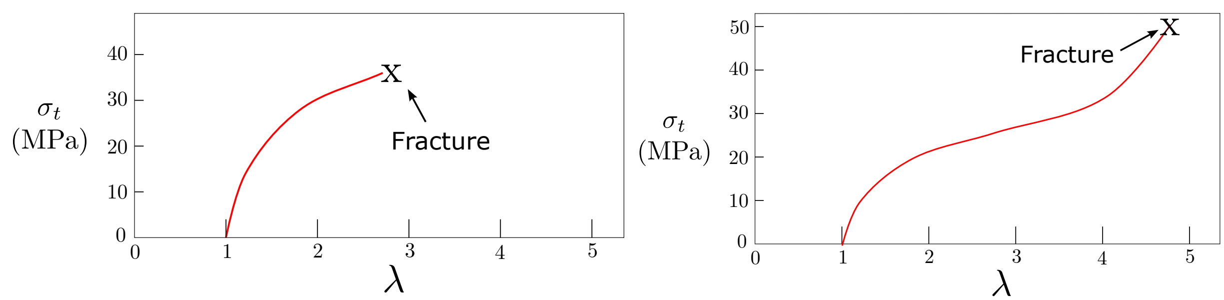
(a) What do you expect for the yield strength of the material in a state of uniaxial compression?

(b) What will the yield strength be under a stress state of pure hydrostatic pressure?

(c) What is the shear yield strength of the material.

15) Consider the following two stress-strain curves obtained from a glassy polymer material. In these plots σ_t is the true stress and λ is the extension ratio ($1+e$, where$e$ is the tensile strain).

(a) (b)

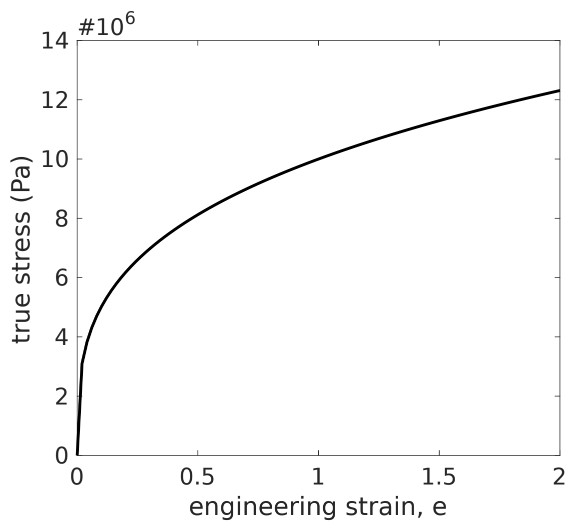


(a) Sketch the behavior of the engineering stress vs extension ratio that you expect for each of these samples in a uniaxial tensile test. Be as quantitative as possible. Briefly describe why you drew the curves the way you did.

(b) Which of these samples can be cold drawn? What value do you expect for the draw ratio? (The plastic strain in the drawn region of the sample)?

(c) Suppose the cross sectional area of each sample is 1 cm^2. What is the maximum load that the sample will be able to support prior to failure for each of the two samples?

16) Consider a material with the following true stress vs. engineering strain behavior, measured in uniaxial extension:



(a) Suppose the cross sectional area of this material is 1 cm^2. Calculate the maximum force that this material would be able to support prior to failure.

(b) Will this material form a stable neck? If so, what is the characteristic strain in the necked region?

17) The following stress tensor describes the state of stress of a material at its yield point:

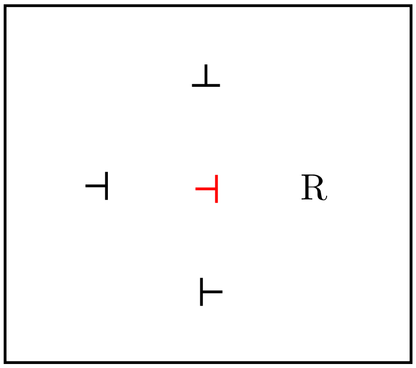
$$\sigma = \begin{bmatrix} 0 & 3 & 0 \\ 3 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix} \mathrm{MPa}$$

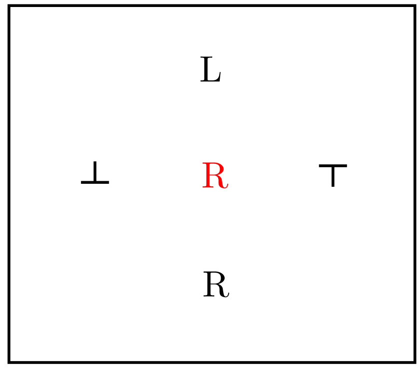
Suppose the same material is subjected to stress state of simple shear. At what value of the applied shear stress do you expect yielding to occur, assuming that the material obeys a Tresca yield criterion.

1.8 Strengthening Mechanisms

18) Consider the two red dislocations at the center of the two diagrams shown below: (All of the dislocations are perpendicular to the plane of the paper.) We are interested in the effect that the central dislocation has on each case on the 4 surrounding black dislocations.

R = right-handed screw dislocation; L = left-handed screw dislocation





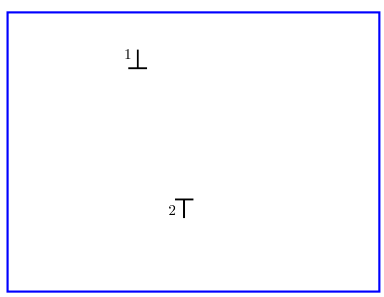
(a) For each of the 5 black edge dislocations, indicate the slip planes with a dashed line.

(b) Draw an arrow on each of the black edge dislocations, showing the direction of the force within its slip plane that is exerted by the red dislocation. If there is no force within the slip plane, circle the black dislocation instead.

(c) For the three black screw dislocations, draw an arrow on them to indicate the direction of the slip force exerted by the red dislocation. If the slip force is zero, circle the black screw dislocation instead.

19) Consider the two edge dislocations shown below. Suppose dislocation 1 remains fixed in place, but that dislocation 2 is able to move on its glide plane.

(a) Assume that the sense vector, $\vec{s}$, for each dislocation is defined so that $\vec{s}$ points into the page. Indicate the direction of $\vec{b}$ for each of the two dislocations.



(b) Indicate the glide plane for dislocation 2 with a dotted line.

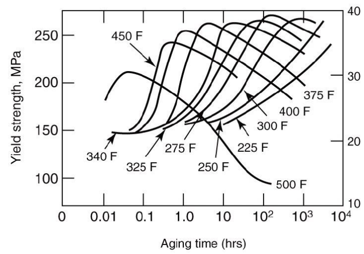
(c) Indicate with an X the location of dislocation 2 at the position within its glide plane that minimizes the total strain energy of the system.

(d) Now suppose that dislocation 1 is a fixed, left-handed screw dislocation and dislocation 2 is a mobile right-handed screw dislocation.

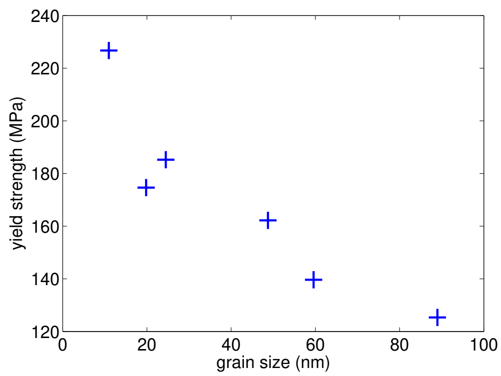
i Use a dotted line to indicate the plane on which you expect dislocation 2 to move in order to minimize the overall strain energy of the system.

ii Plot the overall strain energy of the system as a function of the distance between the two screw dislocations.

20) The figure below shows the yield strength of a precipitation hardened aluminum alloy as a function of aging time at different temperatures. Note that the yield strength initially goes through maximum and then decreases with time. Explain why this happens in as much detail as possible.



21) The following plot shows values of the yield strength of copper samples as a function of the grain size of these samples.



(a) Describe why the yield stress decreases with increasing grain size.

(b) Describe the procedure you would use to determine the limiting value of the yield strength in the absence of grain boundaries.

22) The yield point for a certain plain carbon steel bar is found to be 135 MPa, while a second bar of the same composition yields at 260 MPa. Metallographic analysis shows that the average grain diameter is 50 µm in the first bar and 8 µm in the second bar.

(a) Predict the grain diameter needed to give a yield point of 205 MPa.

(b) If the steel could be fabricated to form a stable grain structure of 500 nm grains, what strength would be predicted?

23) The lattice parameters of Ni and Ni_3Al are 3.52×10^{-10} m and 3.567×10^{-10} m, respectively. The addition of 50 at% Cr to a Ni-Ni_3Al superalloy increases the lattice parameter of the Ni matrix to $3.525 \times$ 10-10 m. Calculate the fractional change in alloy strength associated with the Cr addition, all other things being equal.

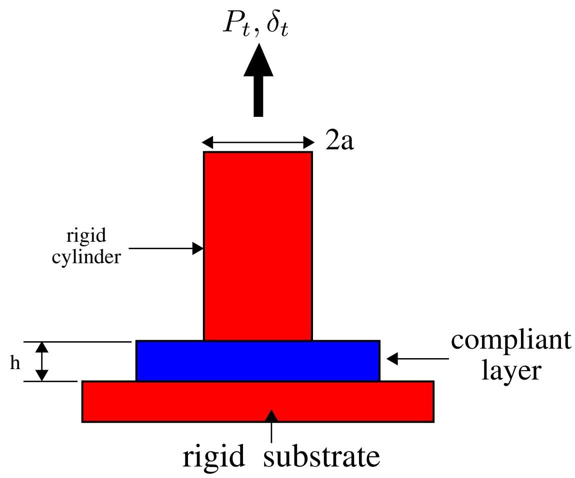
24) General Knowledge

How are G, ν and K related to one another for an isotropic material?

What are typical values of G and K for metals, ceramics and polymers?

1.9 Contact Mechanics

25) Consider the contact of a flat rigid punch with a thin elastic layer, as shown schematically below:



Suppose the compliant layer is incompressible gel ($\nu = 0.5$), with a Young's modulus,E, of 10^4 Pa. The critical energy release rate for failure at the gel/punch interface is 0.1 J/m^2. The punch radius, a, is 3 mm.

(1) What is the tensile force required to separate the punch from the layer if the layer is infinitely thick?

(2) What is the stress intensity factor, K_I, just prior to detachment of the punch from the surface?

(3) How close to the punch edge to you need to be for the tensile stress at the punch/layer interface to be equal to the modulus of the layer?

26) Describe in qualitative terms what happens to the following quantities as the thickness, h, of the compliant layer from the previous problem decreases:

(1) The overall compliance of the system.

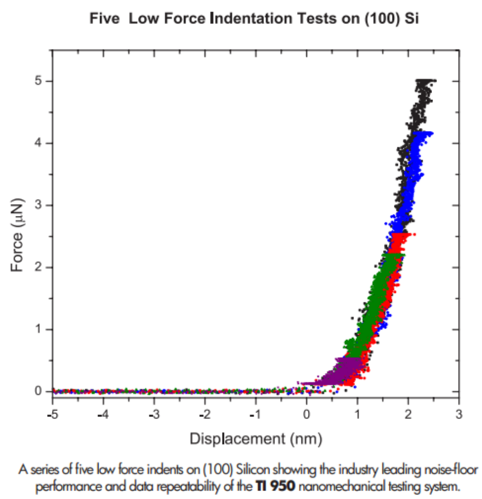
(2) The load required to detach the indenter from the substrate.

(3) The displacement at which the indenter detaches from the substrate.

(4) The shape of the tensile stress distribution at the punch/substrate interface.

1.10 Nanoindentation

27) Commercial nanoindenters are generally not suitable for the characterization of soft materials. To understand why this is the case, consider the following indentation data from the Hysitron web site (this is for the same instrument that Northwestern has in the NUANCE facility):

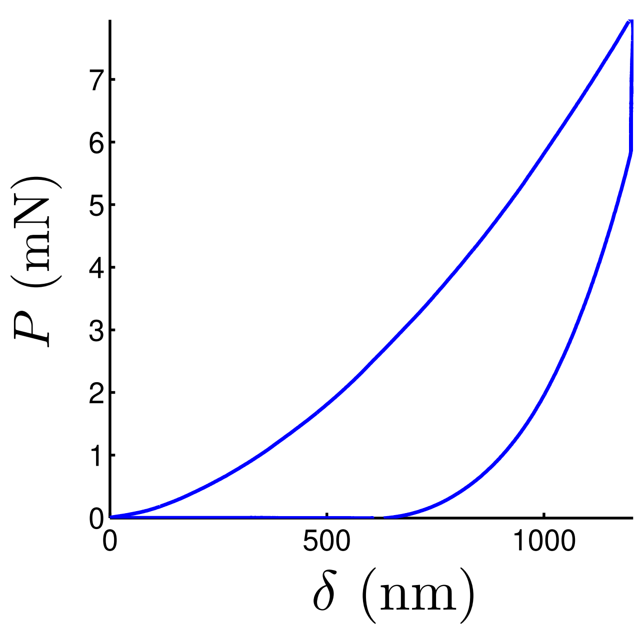


(1) If the data in this figure are obtained with a rigid spherical indenter of radius R, use the data from this figure to estimate the value of R. Assume that the material is being indented elastically and that adhesion can be neglected. (You'll need to look up mechanical property data for silicon).

(2) Suppose that the material is replaced by an elastomer with a modulus of 10^6 Pa. What value of R would need to be used to obtain the same Force displacement curve for this much softer material? (Assume that the effects of adhesion can eliminated by performing the indentation in a suitable liquid).

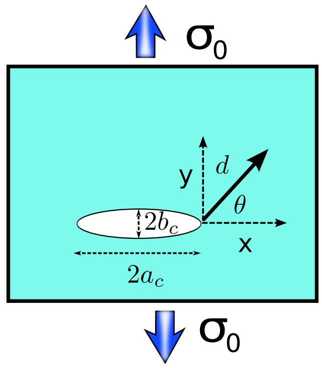
28) Suppose an elastomeric sphere with a radius of 1 mm and a reduced modulus, E^*, of 10^6 Pa is placed on a flat, rigid substrate. Suppose also that the adhesion between the sphere and the substrate is characterized by a critical energy release rate of 0.05 J/m^2, independent of the crack velocity or direction of crack motion. Calculate the radius of the circular contact area that develops between the elastomer and the surface, assuming that there is no external load applied to the sphere (apart from it's weight).

29) Obtain the hardness and elastic modulus from the following nanoindentation curve, obtained from a Berkovich indenter:



1.11 Fracture Mechanics

30) The stress fields in the vicinity of a crack tip in a material are determined by the distance, d, from the crack, and the polar angle θ, defined in the following diagram.



(1) For a fixed value of d, plot the behavior of σ_{xx}, σ_{yy} and σ_{xy} for a mode I crack as a function of θ.

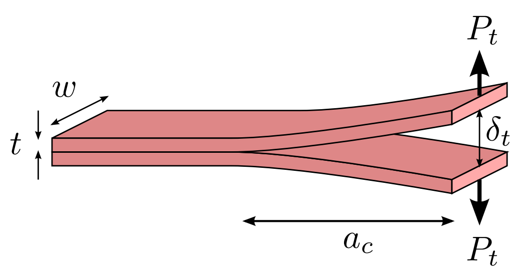
(2) What happens to the stresses for $\theta = 180°$? Why does this make sense?

(3) A mode I crack will travel in the direction for which the normal stress acting across the crack surfaces is maximized. What direction is this?

31) Look up the fracture toughness (K_{IC}) and Young's modulus (E) for window glass. Assuming that the maximum local stress is $\approx E/10$, estimate the crack tip radius of curvature for a crack propagating through window glass.

32) As a crack advances, what happens to the stiffness of the cracked body? What happens to the compliance?

33) A set of double cantilever beam adhesion test specimens was fabricated from a set of aluminum alloy samples. The geometry as as shown below:



Suppose each of the two beams has a thickness (t) of 10 mm, a width (w) of 20 mm and a length of 80 mm. The double cantilever beam sample was produced by using an adhesive to glue the two beams together. Assume the precrack with a length, a_c, of 10 mm. The critical energy release rate for the adhesive is $65\mathrm{J/m^2}$.

(1) Calculate the values of the tensile load, P, and the displacement, Δ, where the crack starts to propagate.

(2) In a load-controlled experiment, P_t is held constant once the crack starts to propagate, and in a displacement controlled test Δ is held constant once the crack starts to propagate. From the relationship between $\mathcal{G}$ and P_t, Δ and a, describe why the load controlled experiment results in unstable crack growth, but the displacement controlled experiment results in stable crack growth.

(3) From your answer to part b, describe how you would design an experiment where you measured the energy release rate required to propagate the crack at a specified velocity.

34) What is crack tip shielding?

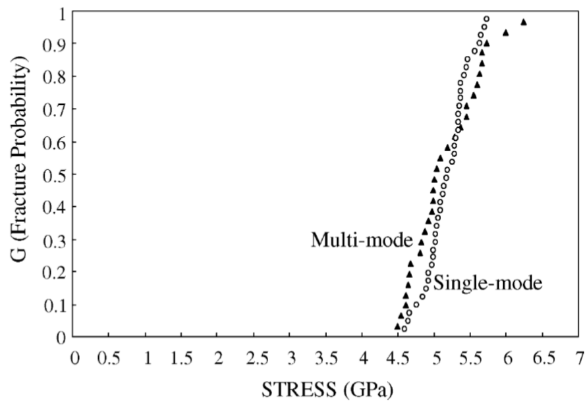
35) Describe the difference between a crack and a craze? How is the maximum width of a craze related to $\mathcal{G}_c$ and K_{IC}?

36) Describe how transformation toughening works to increase the toughness of a ceramic material like ZrO_2.

37) What is a Charpy impact test conducted, and what does it measure?

38) What the difference between the side windows of your car and the windshield? Include the role of tempering, thermal annealing and composite layering, and describe how the desired properties are obtained for the two different applications of glass.

39) The following data were obtained for the fracture stress of a series of silica glass fibers used for optical communications:



The graph shows the distribution of failure probabilities as a function of the applied tensile stress. None of the samples had fractured at a stress of 4.5 GPa, but they had all fractured at a stress of 6 GPa. From these data, and from fracture toughnesses given for inorganic glass in class (and in the course notes), estimate the intrinsic flaw sizes that are present at the surface of the glass fibers. Comment on these sizes, and if you think the fracture mechanics analysis makes sense to use in this case.

40) Silicones containing resin fillers are used as an encapsulant materials in light emitting diodes (LEDs) in order to protect the electronics from harsh environments and to aid in heat dissipation. Near the surface of the electronic components, temperatures can go as high as200 °C for extended time periods.

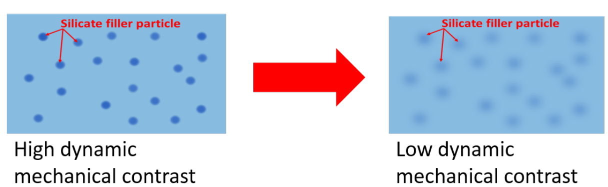


Figure 1.1: High dynamic mechanical contrast is important

(1) Given that a high dynamic mechanical contrast is desirable in creating a soft material with high fracture toughness, what would you suggest as a design criteria in order to maintain high dynamic mechanical contrast at high temperatures? (Hint: think about the role of the T_g of the matrix and filler content.)

(2) Thermal mismatch at the interface between the encapsulant and electronic can lead to residual stresses that promote crack propagation. In assessing the performance of the encapsulant at the interface, should a failure stress or a failure strain criteria be used? Why?

(3) From a thermal management and mechanics perspective, why do you think a soft encapsulant (e.g. silicone) will be more preferable over a hard encapsulant (e.g. glass)?

1.12 Weibull Statistics

41) A set of glass rods with a Weibull modulus of 30 are fractured in a uniaxial tensile test. The stress at which 63% of the samples fracture is 100 MPa.

(1) What is the maximums stress if you want to make sure that less than one in 100 rods fail? (Note that 1/e is 0.37).

(2) What is the maximums stress if you want to make sure that less than one in 10^6 rods fail?

(3) What does the stress need to be to get less than 1 failure in10^6 if the Weibull modulus is 10 instead of 30?

42) What determines the value of the Weibull modulus in a brittle material?

43) A brittle material with a specified geometry fails with a 50% probability at a tensile stress of 100 MPa. From the failure statistics, it is determined that the Weibull modulus for this material is 40. What fraction of these materials will fail at a tensile stress of 70 MPa?

1.13 Viscoelasticity

44) The following questions relate to the DGEBA-PACM/Jeffamine system that was introduced in class.

(1) For the D230-based system, make a plot comparing the temperatures where the slope in $\log(E')$ vs. temperature is maximized, and also the temperature where $\tan(\delta)$ is maximized. Comment on the relationship between these two temperatures.

(2) How many moles of D230 need to be combined with one mole of DGEBA to make a stoichiometric mixture? (no PACM added)

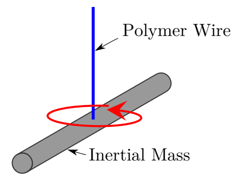
(3) How many grams of D230 need to be combined with one mole of DGEBA to make a stoichiometric mixture? (again assume that no PACM is added. Note that 230 in this case is the molecular weight of the Jeffamine in g/mole. You'll need to calculate or look up the molecular weight of DGEBA to do this DGEBA stands for diglycidyl ether of bisphenol A, but DGEBA is pretty standard abbreviation for it).

(4) What happens to the amount of jeffamine you need to add to get a stoichiometric ratio as the molecular weight of the jeffamine is increased as you move from D230 to D400 to D2000 to D4000? (a qualitative answer is okay - you don't need to be quantitative for this. Continue to assume that no PACM is added).

(5) What happens to the glass transition temperature for samples without any PACM as the molecular weight of the Jeffamine increases from 230g/mole to 400 g/mole? Describe how you obtained T_g from the data shown in the lecture. Also describe why the trend in T_g is as you describe.

(6) Mixtures with DGEBA and an equal amount of jeffamine and PACM become cloudy as the molecular weight of the jeffamine increases. Why is this?

45) Consider a cylindrical metal bar with a density of 7.6 g/cm^3, a diameter of 1 cm and a length of 10 cm. It is suspended from a polymer fiber with a length, ℓ, of 30 cm and a diameter of 1 mm.



(1) Suppose the fiber is perfectly elastic, with a shear modulus 10^9 Pa. Calculate the natural frequency of the system if the bar is rotating back and forth, causing the fiber to twist.

(2) Suppose the fiber is viscoelastic, with G' at the frequency calculated from part a equal to 10^9 Pa, and with $G'' = 10^7$ Pa. How many periods of the oscillation will take place before the magnitude of the oscillation decays by a factor of e (2.72)? Note: the rotational moment of inertia for the suspended metal bar in this geometry is $m\ell^2/12$, where m is the total mass of the bar and ℓ is its length.

46) As mentioned in class, the Maxwell model, with a viscous element and an elastic element in series with one another, is the simplest possible model for a material that transitions from solid-like behavior at short times, to liquid-like behavior at long times. For a shear geometry we refer to the elastic element as G_0 and the viscous element as η.

(1) For what angular frequency are the storage and loss moduli equal to one another for a material that conforms to the Maxwell model? Express you answer in terms of the relaxation time, τ.

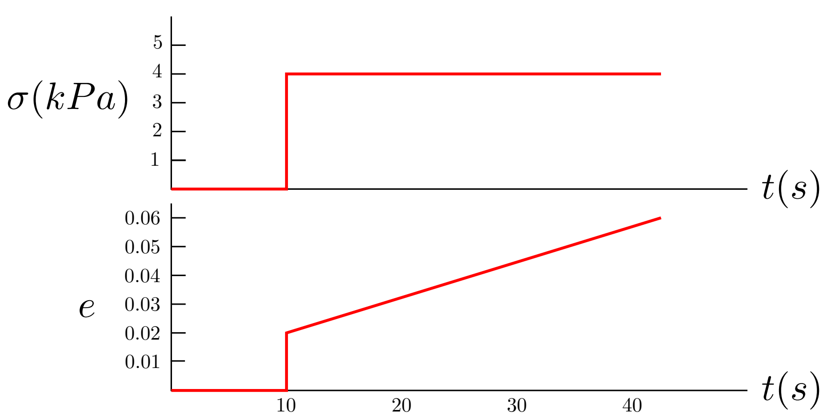
(2) Suppose the material is oscillated at a frequency that is ten times the frequency you calculated from part a. Does the material act more like a liquid or a solid at this frequency? Describe your reasoning.

(3) Calculate the values of G' and G'' at the frequency from part b, and calculate the phase angle, ϕ describing the phase lag between stress and strain in an oscillatory experiment. Note that the following expression relates ϕ, G' and G'':

$$\tan\phi = \frac{G''}{G'}$$

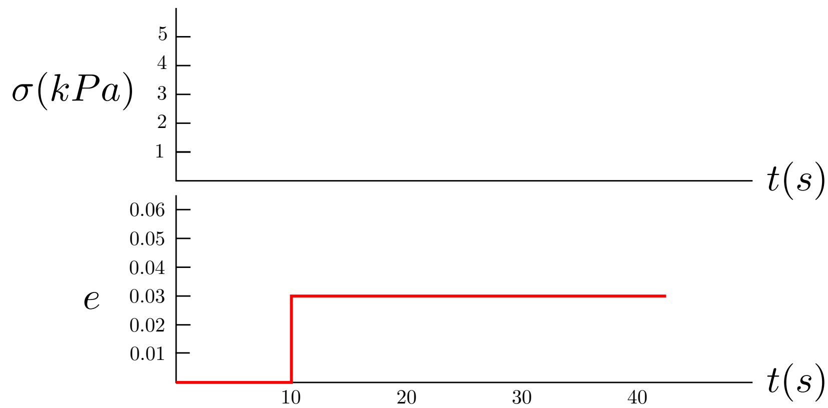
Does this phase angle make sense, given your answer to part b? Compare your value of ϕ to the values you expect for a perfectly elastic solid and a perfect liquid.

47) The following stress and strain response are observed for a material during the initial stages of a creep experiment.

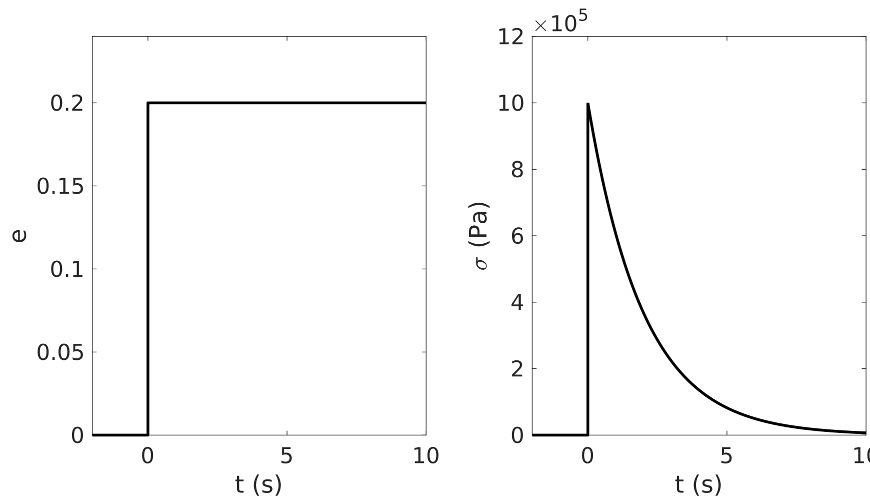


(1) Draw a spring/dashpot model that describes this behavior. Label moduli and viscosities as quantitatively as possible.

(2) A stress relaxation test (strain shown below) is performed on the same material. On the stress axis below, draw the stress response that you expect for the model you have drawn from part a.



48) A tensile experiment is performed on a viscoelastic material, with the tensile strain (e) and tensile stress (σ) exhibiting the time dependence shown in the following figure:



(1) Draw a spring and dashpot model that would give this response. Give values (modulus or viscosity) for each element in your model (the values of these quantities are not expected to be exact).

(2) Suppose the sample were vibrated in tension at a frequency of 1000 Hz (cycles per second). Estimate the value of $|E^*|$ (magnitude of the complex shear modulus) that you would expect to obtain.

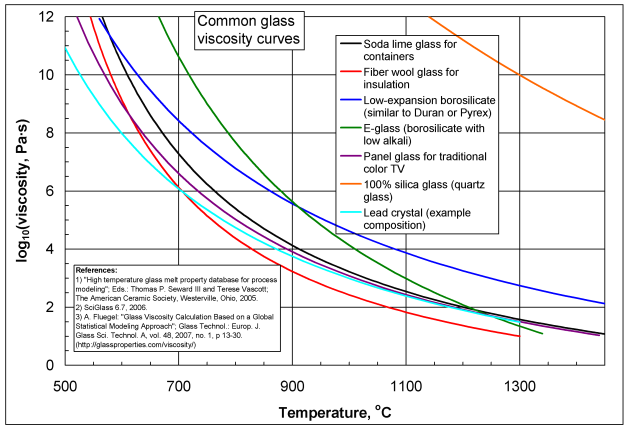
(3) For what range of frequencies do you expect the loss modulus (E'') to exceed the storage modulus (E') for this material?

49) Can creep of a glass window by viscous flow give measurable changes in sample dimensions over a very long period of time? To sort this out, do the following:

(1) Estimate the stress at the bottom of a very tall pane of window glass, due to the weight of the window itself. Look up the density of silica glass, and a height of the window that makes sense (choose a big one).

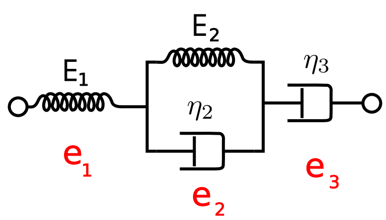
(2) Estimate the viscosity that would be needed to give a measurable change in sample dimensions after 400 years.

(3) Using the data below, does it make sense to you that observable flow could noticeably change the dimensions of the window? (You'll need to make some assumptions about how the viscosity will extrapolate to room temperature.



1.14 Nonlinear Viscoelasticity and Creep

50) A step stress (0 for t<0, σ for t>0) is applied to a solid which can be modeled by the following combination of linear springs and dashpots:



(1) This model is useful because it includes a non-recoverable creep component, a recoverable time dependent creep component, and an instantaneous, recoverable strain.

a) Identify the element (or combination of elements) in the above model which is associated with each of these three contributions to the strain.

b) Write down the expression for the total strain, $e(t)$, after the imposition of the step increase in stress.

c) Suppose the stress is applied for a long time, so that the strain is increasing linearly with time. At some time, t', the stress is removed. Derive an expression for the change in strain after the stress is removed.

(2) This model has been applied to creep data for oriented polyethylene at room temperature. Model predictions were compared to data obtained from samples of high molecular weight (High M) and low molecular weight (Low M). Both samples were drawn to the same draw ratio. The following values of E_1, E_2, η_2 and η_3 were obtained from experimental data:

Sample	σ (GPa)	E_1 (GPa)	E_2 (GPa)	η_3 (GPa-s)	η_2 (GPa-s)
Low M	0.025	17.4	33.5	$1.8\text{x}10^5$	4300
Low M	0.05	13.6	35.6	$6.3\text{x}10^4$	5000
Low M	0.1	17.7	26.4	$3.1\text{x}10^4$	2200
Low M	0.15	17.7	26.5	$2.6\text{x}10^4$	2300
Low M	0.2	16.4	26.8	$1.2\text{x}10^4$	2000
High M	0.1	18.3	31.9	$3.1\text{x}10^6$	$8.7\text{x}10^4$
High M	0.15	16.6	21.3	$1.7\text{x}10^6$	$7.3\text{x}10^4$
High M	0.2	15.8	32.7	$7.7\text{x}10^5$	$3\text{x}10^4$
High M	0.3	25.4	39.1	$4.8\text{x}10^4$	2800
High M	0.4	25.0	43	$3\text{x}10^4$	3000
High M	0.5	21.7	46	$2.5\text{x}10^4$	5000

From the values of η_3 given in this table, determine the stress dependence of the steady state creep rate. From this stress dependence, calculate the activation volume for non-recoverable creep in the high and low molecular weight samples, and compare these values to one another.

51) Creep in metals at low stresses occurs by a vacancy diffusion mechanism, which means that the activation volume for these creep mechanisms corresponds to the atomic volume. Show using the data below for silver that we can safely replacesinh $(\sigma v/2k_BT)$ with $\sigma v/2k_BT$, so that the creep rates are linear in stress at all relevant temperatures and stresses where the dominant creep mechanisms involve vacancy diffusion. (You'll need to look up data you can use to calculate the atomic volume of silver).